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motion, as translated by Sherburne (*Manilius*, p. 210), are :—"Its motion was constantly retrograde, from South to North, by the Hare, Foot of *Orion*, *Taurus*, to the *Pleiades*; and from thence as far as *Perseus*; not directly but obliquely from East South East to North North West, in a great circle inclined to the Ecliptick, and Æquator: At first by its diurnal motion compleating 11° 16′, at last 30′, it ran through in the time of its duration 65° 51′ passing beyond and beside the course of the Annual Orb."

Hevelius declares positively that the comet disappeared after January 10. His words are: "Quamobrem serio sum persuasus ultimo hâc die esse visum; et postmodum à nemine observatum esse: quemadmodum et mihi, die 11 Januarii, nusquam apparuit." There is a further marginal note: "Nullus, præter autorem, cometam ad 11 usque Januarii observavit."

The observations of December 21, 26, 27, 28, 29, 30, and January 3, which state that the "latitude" of the comet was 3°, 37°, 40°, $42\frac{1}{2}$°, 45°, 47°, and $51\frac{1}{2}$° respectively, must not be taken as the latitude from the ecliptic; but they are what Hevelius terms "Distantia ab intersectione Æquatoris"—that is to say, the angular distance of the comet from the equator in its apparent orbit considered as a great circle.*

On the Straightness of Spider Lines.

By H. H. Turner, M.A., B.Sc., Savilian Professor.

1. The gist of the following note is that the horizontal wire of the Greenwich Transit Circle is sensibly straight. I was led to examine this point from hearing that a curvature or "sag" of the horizontal wire had been detected elsewhere. If such a curvature existed it would specially affect the R—D observations, and might form a considerable part of this curious phenomenon.

2. The method adopted was to take from the section of observed zenith-distances in the volumes of Greenwich Observations for 1892 and 1893 all those cases in which a star had been observed in zenith-distance at the vertical wires 1, 2, 6, 7. The readings of the micrometer at wires 1 and 7 were subtracted

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* In the copy of Bayer under consideration, the author of the observations has designated the group of stars 33, 35, 39, and 41 *Arietis* as "Nubes *Tauri* vel Halitus *Arietis*." These stars formed the constellation "Musca," or "Apes," of Jacobus Bartchius, the son-in-law of Kepler, and I am unaware of any other designation for them than his. The explanation of "Nubes *Tauri*" is that the stars are situated on the confines of the cloud which conceals the body of the Bull, as shown in the old drawings of the constellations in Aratus and Hyginus, and in Bayer. "Halitus *Arietis*" is explained by the face of the Ram being turned in the direction of these stars, and thus the animal's breath would cover them.

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from those at wires 2 and 6. If the path of the star across the field were a straight line, and there were any "sag" of the wire, the result (2+6)—(1+7) would be positive. The inclination of the wire to the horizontal would be eliminated.

3. The path of a star is, however, curved, and the correction for curvature is given in the Greenwich Observations as

$$+0^{r}{\cdot}00327 \times \cot \text{N.P.D.} \times (\text{wire}-4)^2,$$

the unit 1^r being a revolution of the telescope micrometer, $=37''{\cdot}05$, so that $^r{\cdot}00327=0''{\cdot}121$ (it is convenient to keep our figures in revolutions of the micrometer). Thus the correction to the quantity tabulated (2+6)—(1+7) is

$$+\{(2^2+2^2)-(3^2+3^2)\} \times {}^r{\cdot}00327 \times \cot \text{N.P.D.}$$

or

$$-{}^r{\cdot}0327 \times \cot \text{N.P.D.},$$

which was accordingly applied to the results.

4. In the following table the results are given, arranged according to N.P.D. The approximate mean N.P.D. of the group is given in the *first* column, each group including all the stars within 10° of N.P.D. In the *second* column is given the curvature correction corresponding to this mean N.P.D., to give an idea of its magnitude. The observations were, however, each corrected separately, and not by use of this mean correction. In the *third* column is given the flexure factor sin Z.D. It seems probable that if there were any sag in the wire, it would be greater at large zenith-distances, according to some law like that of flexure.

N.P.D.	Correction for Curvature.	Factor for Flexure.	Readings at wires (2+6)—(1+7); corrected for curvature.							
			1892_1.	No. Obs.	1892_2.		1893.		Mean.	
°	r		r		r		r		r	
45	−0·033	0·10	−·015	8	−·016	6	−·002	7	−·011	21
55	−0·022	0·28	−·003	14	−·001	6	−·011	13	−·006	33
65	−0·015	0·44	−·002	37	−·002	10	+·001	18	−·001	65
75	−0·009	0·59	−·004	20	−·004	16	+·004	17	−·001	53
85	−0·003	0·72	+·016	9	+·005	12	+·001	11	+·007	32
95	+0·003	0·83	−·002	12	+·007	14	+·001	9	+·002	35
105	+0·009	0·91	+·001	10	+·011	8	+·006	22	+·006	40
115	+0·015	0·97	+·003	6	−·007	19	−·006	13	−·005	38
123	+0·022	1·00	+·048	2	−·004	4	−·018	9	−·005	15

In columns 4, 6, 8, are given the mean results of three series of groups, corresponding to the dates 1892 January 1–June 30, 1892 July 1–December 31, 1893 January 1–December 31: the number of stars in each group being shown in columns 5, 7, and 9. Columns 10 and 11 give the weighted means (according to number of observations) and weights for all the stars.

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5. There appears to be no evidence of curvature in the wire. The simple mean of column 10 is $-^{r}{\cdot}0016$, and the weighted mean $-^{r}{\cdot}0008$; equivalent to $-0''{\cdot}06$ and $-0''{\cdot}03$ respectively; and indicating an *upward* curvature if anything. If we equate the results of column 10 to k sin Z.D., and solve by least squares, we get

$$k = -^{r}{\cdot}0003 = -''{\cdot}01.$$

6. The negative value for 45° is noteworthy, especially as the few available observations on the north side of the zenith also give negative results, as follows—

°	r	°	r	°	r	°	r
−45	−·018	−36	−·051	+25	−·082	+28	−·009
−42	−·042	−35	+·007	+25	−·032	+35	−·011

But there are not enough observations to settle anything definitely about this side of the zenith. There is one way in which such negative results for stars above the pole may be spurious, viz. if the observer inadvertently transposes the readings of the micrometer at wires 1 and 2, or at 6 and 7. Such errors would give negative results in the above investigation.

7. It occurred to me as possible that the law assumed for curvature of the star's path might not be sufficiently accurate near the pole; but a very little examination showed that it is accurate enough except for close polar stars, which are not usually observed far from the meridian at Greenwich. I take this opportunity of pointing out another case where the use of rectangular coordinates simplifies the problem very much.

8. The accurate formula for correcting for curvature is given by Chauvenet (vol. ii. p. 291) as

$$-\frac{\sin^2\frac{1}{2}(\tau-m)}{\sin 1''}\,.\,\sin 2\delta' - \frac{2\sin^4\frac{1}{2}(\tau-m)}{\sin 1''}\sin 2\delta' \sin^2\delta', \&c.$$

where τ is the east hour angle of the star, and c, m and n are the usual instrumental constants. It will be noted that c and n do not appear in this expression. If we use rectangular coordinates x and y along the horizontal and vertical wires respectively; then, since the path of the star is an ellipse (the projection of a small circle on a plane) its equation is

$$y = ax^2 + by^2.$$

To find a and b, perhaps the simplest plan is to take three rectangular axes S O, S P, and a perpendicular to both; where O is the centre of the celestial sphere, O P the polar axis, S the star, and S P a tangent to the sphere at S. The coordinates of O are (0, 0 1) if S P be taken as axis of y and S O of z; and the coordinates of P are (0, tan p, 0); so that the direction cosines of O P are proportional to (0, tan p, -1). Let (x, y) be any point

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Q of the star's path traced on the tangent plane. The direction cosines of O Q are proportional to $(x, y, -1)$; and thus, since this line is inclined to O P at angle p we have

$$\cos p = \frac{y \tan p + 1}{\sqrt{(1 + \tan^2 p)(1 + x^2 + y^2)}},$$

or

$$1 + x^2 + y^2 = 1 + 2y \tan p + y^2 \tan^2 p$$

or

$$y = \tfrac{1}{2} \cot p \,.\, x^2 + \cot 2p \,.\, y^2.$$

When p is small, this may be written

$$y = \frac{x^2}{2p} + \frac{x^4}{8p^3} = x \left[\left(\frac{x}{2p} \right) + \left(\frac{x}{2p} \right)^3 \right]$$

all the quantities being expressed in circular measure.

The distances of wires 1 and 7 from the meridian at Greenwich are about $11'$; so that for a star 1° from the pole $\dfrac{x}{2p} = {\cdot}09$, and $\left(\dfrac{x}{2p}\right)^3 = {\cdot}0007$; thus the second term produces an effect of $11 \times 60 \times {\cdot}0007 = 0''{\cdot}5$, which is quite appreciable. But at 2° from the pole this becomes $0''{\cdot}06$; so that it is negligible except for close polar stars.

9. We have in the above neglected the instrumental constants c, m, n, and the inclination of the wire. The effect of the constants c, m, n is to shift the apparent origin of coordinates to the right by the quantity

$$(c + m \sin p + n \cos p) \equiv c' \text{ say},$$

assuming that c, m, n are expressed in seconds of arc, and thus we have

$$y = \tfrac{1}{2}(x - c')^2 \cot p + y^2 \cot 2p.$$

But c' is very small. The chief additional term is $-c'x \cot p$, which is eliminated in symmetrical transits (at wires 1 and 7 or 2 and 6 say), and does not therefore affect the above investigation concerning $(2+6)-(1+7)$ for instance. The constant term $\tfrac{1}{2}c'^2 \cot p$ is very small indeed, and in any case does not affect the above investigation.

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On the Apparent Disc and on the Shadow of an Ellipsoid.

By A. Marth.

If k denotes the equatorial radius and $e = \sin \epsilon_0$ the eccentricity of a planet's ellipsoidal surface, so that the polar radius is $k \cos \epsilon_0$, the outline of the planet's disc, as seen from a point in planetocentric latitude B above the plane of the planet's equator and at the distance Δ, will be an ellipse, the semi-axes a and b of which are

$$a = \frac{k}{\Delta}\omega'' \qquad \omega'' = \frac{1}{\text{arc }1''},$$

$$\begin{aligned} b &= a\sqrt{1 - e^2 \cos^2 \mathrm{B}} \\ &= a \cos \epsilon, \end{aligned}$$

If, further,

$$\left.\begin{aligned} \sin \epsilon &= \sin \epsilon_0 \cos \mathrm{B}. \\ \cos \epsilon \sin \nu &= \sin \epsilon_0 \sin \mathrm{B} \\ \cos \epsilon \cos \nu &= \cos \epsilon_0, \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (1)$$

the proportion $\cos \epsilon$ of the axes of the apparent disc is found from the proportion $\cos \epsilon_0$ of the axes of the ellipsoid by

$$\cos \epsilon = \cos \epsilon_0 \sec \nu, \text{ where } \tan \nu = \tan \epsilon_0 \sin \mathrm{B},$$

and $\cos \epsilon_0$ from $\cos \epsilon$ by

$$\cos \epsilon_0 = \cos \epsilon \cos \nu, \text{ where } \sin \nu = \tan \epsilon \tan \mathrm{B}.$$

These simple formulæ for the outline of the apparent disc are only valid when the distance Δ in comparison with k is so great that terms of the second order $\frac{k^2}{\Delta} \dots$ may be neglected. If that is not the case, strict formulæ must be substituted.

Let λ and β denote the planetocentric longitude and latitude, referred to the plane of the planet's equator, of a point on the ellipsoidal surface, $k\rho$ its linear distance from the centre of the ellipsoid, and L B Δ the corresponding coordinates of the point of view, so that L B and $k \cos \nu$ are the coordinates of the centre point of the apparent disc. Further, let $p - \mathrm{P} = \pi$ denote the position angle of the surface point at the apparent centre of the disc, reckoned from the direction P of the polar axis, σ the planetocentric angular distance or the angle between $k\rho$ and Δ, Δ' the linear distance between the surface point and the point of view, and s the angle between Δ' and Δ. The values of π σ s Δ' are found by

$$\left.\begin{aligned} \sin \sigma \sin \pi &= \cos \beta \sin (\mathrm{L} - \lambda) \\ \sin \sigma \cos \pi &= -\cos \beta \cos (\mathrm{L} - \lambda) \sin \mathrm{B} + \sin \beta \cos \mathrm{B} \\ \cos \sigma &= \cos \beta \cos (\mathrm{L} - \lambda) \cos \mathrm{B} + \sin \beta \sin \mathrm{B} \\ \Delta' \sin s &= k\rho \sin \sigma \\ \Delta' \cos s &= \Delta - k\rho \cos \sigma. \end{aligned}\right\} \quad \dots \quad (2)$$